

# Chapter 2

## Background and Context

*In this chapter, we provide a cursory overview of the particle physics enterprise as a whole and touch on the relevance of the particular work described in this thesis. Setting the stage in this fashion requires the introduction of new pieces and characters, and a context in which they seem real. Rather than developing the background in excruciating detail, however, we will concentrate on the objects in the foreground that are most essential to the story line. If the subsequent narrative is less than compelling, blame the author for not seeding the tale with the right elements in the beginning, and try to find those passages that still ring authentic and true.*

### 2.1 Introduction

Experimental work often takes place in the context of some model, picture, or other kind of understanding about the actual physics that drives a particular process. The experiment then typically extends the validity of the theory by confirming some previously made prediction. Or perhaps the experiment reveals some unforeseen behavior that requires re-interpretation of the theory or even the construction of a new one. In either case, the theory provides a prediction which is then confirmed or refuted by experiment.

In this particular project, however, the experiment–theory connection is used in a slightly different way. Although particle physicists have an incredibly successful and well-tested theory of matter at the smallest scales (called the Standard Model), there are still certain calculations—and so predictions—that lie beyond the reach of our current computational tools. Such is the case with the charmless, semileptonic decay of the  $b$  quark, written schematically as  $b \rightarrow u \ell \nu$ . A difficult, tedious, and ultimately only approximate calculation can be made to predict the frequency with which this decay occurs, but even more work is required if one desires quantitative details about the various particles in the aftermath of the quark-level decay. The kinematic properties of those daughters, such as the spectrum of energies of the charged lepton, or the range of masses of the hadronic system that forms around the  $u$  quark, are, quite bluntly, beyond reach of calculation. Both of these examples are instances where a single, indisputable theoretical prediction simply cannot be made because the phenomenological computation cannot be broken into pieces that are calculable. Instead, modern phenomenologists are reduced to looking for connections between this particular decay process and other decays of the  $b$  quark. By applying powerful mathematical analogies, they can transform the experimental

measurement of a different  $b$  decay process into a prediction about the  $b \rightarrow u \ell \nu$  decay. The work described in this thesis is a small example of such a step taken in this “backwards” direction: data is used to essentially “create” a prediction in an area where traditional theory is silent.

The relevance of this particular decay (and so this work) lies in its great utility in measuring a fundamental parameter of the Standard Model called  $|V_{ub}|$ . This number controls how often a  $b$  quark will decay weakly into a  $u$  quark and the  $W$  particle (described by the reaction equation  $b \rightarrow uW$ ), but it plays a more important role in our understanding of the balance between matter and antimatter in the universe today. The fact that this particular decay happens at all (*i.e.* that  $|V_{ub}| \neq 0$ ) makes the Standard Model more interesting because it allows for a mechanism generically called “CP violation,” one of the necessary ingredients in any story explaining why the universe seems to have so much matter—and so little antimatter—today. By measuring this key input to the Standard Model, we can better understand the amount of CP violation accommodated by the model, and so help answer the question of whether the Standard Model mechanisms can possibly account for the large imbalance—or asymmetry—that we observe today.

The weight of evidence is already against that possibility: it seems unlikely that the Standard Model can explain the enormous excess of matter over antimatter. In fact, it is clear that the Standard Model doesn’t even apply to the extreme energies, densities, and temperatures of the early, evolving universe.<sup>1</sup> What theory, then, does apply? By more deeply exploring the data to which we have access in present-day particle accelerators, we hope to find an answer. There are clues all around that point toward an extension or replacement of the Standard Model that can explain—even predict—the appearance of the universe today, but the challenge is to find enough of these clues to see, even if dimly, what this new picture of the physical world will be. One exciting place to begin this search is in the weak decays of quarks, and in the decay of the  $b$  quark in particular, since we have good reason to believe that the new physics will show up quite clearly in this arena. Naturally, we have to be clear about what it is we already do understand so that we can identify the data that don’t fit—the clues that can point the way to the future theory. In another context, then, the work described in this thesis is one small step on that road to pinning down what we do know, pressing against the boundaries of what we don’t.

The crux of the computational difficulties surrounding  $b \rightarrow u \ell \nu$  is the fact that the  $b$  quark is never found in isolation in nature. In our context, it is always embedded in a larger particle called the  $B$  meson. There the  $b$  quark is paired with another (anti-)quark in a bound-state that greatly complicates the simple picture of the  $b \rightarrow uW$  transition. The quark and its decay cannot be studied on its own;

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<sup>1</sup>The Standard Model is often abused as well for its lack of explanatory power, a matter discussed below. Among other things, it doesn’t explain the patterns (and patterns within patterns) of the masses, charges, and couplings of the elementary particles.

it must always be disentangled from the arms of both its partner quark in the meson and an ever-present but hazy cloud of other “virtual” particles that blurs the definition of exactly what and where the  $b$  quark itself really is.

So, in the laboratory, we are limited to study of the broader category of  $B$  meson decays, and must cope with all of the complications this implies. Some decays do occur via the  $b \rightarrow u \ell \nu$  channel at the quark level, and identifying these decays is itself a challenge. But there are other difficulties as well: namely, other  $B$  decay processes can mimic the  $b \rightarrow u \ell \nu$  decay and frustrate efforts to make a clean measurement of the decay rate and so eventually extract an estimate of  $|V_{ub}|$ . Another difficulty is our limited ability to predict the detailed dynamics of  $b \rightarrow u \ell \nu$  decay. Any technique more sophisticated than a naïve free-quark calculation must somehow account for the implicit strong dynamics that envelops the quarks, even if the calculation is built from terms combining the quark fields. The current framework for analyzing  $b \rightarrow u \ell \nu$  provides a formal structure for understanding the decay in such a fashion, but practical and theoretical limitations require simplifying the general analysis to just a few terms. One of the neglected terms corresponds to a theoretical mechanism known as “weak annihilation,” a process in some ways analogous to the direct annihilation of the two quarks in a meson into the carrier of the weak force, the  $W$  boson.

Compared to the total  $b \rightarrow u \ell \nu$  rate, the contribution of weak annihilation is expected to be small, but complete description of its kinematics—*where* it contributes—is notoriously difficult to evaluate. The significance of weak annihilation lies in what little is known about its relative importance across the full phase space of possible  $B \rightarrow X_u \ell \nu$  decays. In analogy to the monochromatic purely leptonic decay of a meson, the contribution is expected to be effectively limited to a small region of phase space, and for experimental reasons we will explore shortly, it is in this same region that most observations of  $b \rightarrow u \ell \nu$  are made. Thus experimental measurements that employ a truncated rate calculation will inevitably neglect the presence of a feature of the decay to which they may be acutely sensitive.

A complete understanding of the  $b \rightarrow u \ell \nu$  rate requires knowing the relative size of the weak annihilation term(s) in comparison to the few leading terms retained in the phenomenological calculation. A correct decomposition is clearly essential to interpreting experimental measurements of  $b \rightarrow u \ell \nu$ , since some portion of the observed, true rate arises from the weak annihilation graph but is not accounted for in current calculations. Our ignorance of weak annihilation thus directly limits our understanding of  $b \rightarrow u \ell \nu$ , and so the precision with which we can measure  $|V_{ub}|$ . This can ultimately derail our hopes to test the Standard Model as a viable explanation for why the universe appears as it does today.

The goal of this analysis is to use the data we do have to “inform the theory,” to constrain our ignorance of weak annihilation with an experimental measurement. We try to answer the questions: *Just how big can this weak annihilation process*

*be? Given that we know (approximately) where it should occur, can we see it? Do we see it? Can we use data to help determine or at least constrain the details of this process? Can we learn something from the data even in the absence of a robust prediction for weak annihilation?*

The key to this puzzle is that even though we don't have a concrete description of the detailed physics of weak annihilation,<sup>2</sup> we have a good handle on where it is important in the the space of  $B$  decay configurations, and where it is not. By carefully comparing the data in different regions against itself, we can actually derive a data-driven limit on just how large this unknown process can be, and thus a limit on how much it can impact standard  $b \rightarrow u \ell \nu$  measurements. More broadly, our knowledge of  $b \rightarrow u \ell \nu$  and the related  $b \rightarrow c \ell \nu$  decays has seen significant progress in just the last few years. Such excellent understanding of the semileptonic  $B$  decay process as a whole, combined with a careful scrutiny of the data, allows us to tease out some valuable information to feed back into the broader theoretical picture of the general  $b \rightarrow u$  transition.

With these preliminaries out of the way, let us now turn to a more careful survey of the backdrop for the analysis.

## 2.2 The Standard Model

The name “Standard Model” is the label applied to the wildly successful picture that particle physicists have developed and refined over the past fifty years to explain the content and interactions of matter at ridiculously tiny distance scales and enormously high energies. What concerns us here is only the empirical content of the theory<sup>3</sup> and a survey of the features of the Standard Model's design that lie behind its success. (For a more in-depth review of the Standard Model, including its development, history, and limitations, consult any of Refs [1, 7–9, 16] or the references listed therein.)

### 2.2.1 Quarks and Leptons

The Standard Model posits that all matter is made of particles known as fermions, which are divided into the two categories of quarks and leptons. The essential distinction between the two types will become clear below. Fig 2.1 provides some basic information about these elementary particles, listing their names, electric charge, and masses, as well as indicating a grouping into “families” that

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<sup>2</sup>The ignorance here is best described as two-fold: theorists can't say much about the process, and I can't understand even what little they do say.

<sup>3</sup>Let's not digress with a discussion of the distinctions between a theory and a model. Whoops, I guess we almost did, anyway. . . .

reflects a pattern in nature that the theory itself cannot explain in any deep way. The canonical picture is that these particles are truly fundamental, indivisible, and point-like.<sup>4</sup> The particle physics enterprise for the last several decades has been directed toward first identifying these particles, and then understanding the relationships between them. That assembled knowledge is fully captured in this simple list of what it is that constitutes the observable “stuff” in the universe.<sup>5</sup>

<b>FERMIONS</b>			matter constituents spin = 1/2, 3/2, 5/2, ...		
Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c <sup>2</sup>	Electric charge	Flavor	Approx. Mass GeV/c <sup>2</sup>	Electric charge
$\nu_e$ electron neutrino	$<1 \times 10^{-8}$	0	<b>U</b> up	0.003	2/3
<b>e</b> electron	0.000511	-1	<b>d</b> down	0.006	-1/3
$\nu_\mu$ muon neutrino	<0.0002	0	<b>C</b> charm	1.3	2/3
<b><math>\mu</math></b> muon	0.106	-1	<b>S</b> strange	0.1	-1/3
$\nu_\tau$ tau neutrino	<0.02	0	<b>t</b> top	175	2/3
<b><math>\tau</math></b> tau	1.7771	-1	<b>b</b> bottom	4.3	-1/3

**Figure 2.1:** The fermions of the Standard Model. Quarks (which can interact via the strong interaction) appear on the right, leptons (which do not) on the left. Essentially all of the naturally-occurring matter on our planet (and in the universe) is composed from only two kinds of quarks ( $u$  and  $d$ ) and one kind of lepton ( $e$ ). These three particles combine to form atoms, the chemical building blocks of all matter. Figure reproduced by permission [10].

These universal ingredients<sup>6</sup> do not exist idly on their own, unaware of the existence of other particles. They can influence each other through a set of equally fundamental interactions that make up the rest of the Standard Model.

<sup>4</sup>As we shall see later, however, this description can be viewed as nothing more than a convenient fiction that applies under certain circumstances, with no necessary ontological commitment.

<sup>5</sup>The word “observable” is used to qualify this claim since it is now clear that the universe consists largely of so-called dark energy and dark matter, new forms of matter-energy not accommodated by the Standard Model. These phenomena are described as “dark” precisely because they are not directly observable, yet actually seem to dominate the composition of the universe today.

<sup>6</sup>Pun intended. Stay tuned. Later, there’s a joke about a traveling salesman and the farmer’s daughter.

## 2.2.2 Interactions

The interactions between the particles in the Standard Model can all be described as the result of only four basic forces, described briefly in Fig 2.2. Gravity is the weakest of the four, and is ignored in the particle physics setting because the masses of the elementary particles are so small. Its effects at these scales are always dwarfed (by many orders of magnitude) by those of the other forces.

PROPERTIES OF THE INTERACTIONS					
Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic	Strong	
				Fundamental	Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	$W^+$ $W^-$ $Z^0$	$\gamma$	Gluons	Mesons
Strength relative to electromag for two u quarks at: for two protons in nucleus	$10^{-41}$ $10^{-41}$ $10^{-36}$	0.8 $10^{-4}$ $10^{-7}$	1 1 1	25 60 Not applicable to hadrons	Not applicable to quarks 20

**Figure 2.2:** The four fundamental interactions in the Standard Model. Their relative importance at the subatomic scale is shown, making it clear why gravity is never considered when analyzing the behavior of elementary particles. Figure reproduced by permission [10].

Due to the short distances across which Standard Model particles typically interact ( $10^{-10}$ – $10^{-15}$  m), they must be treated with the tools of quantum field theory, the combination of special relativity and quantum mechanics appropriate for this regime of high energies and small distances. In field theories, interactions are described not with mysterious action-at-a-distance forces, but as the direct result of some force-carrying particle impacting the particle being acted upon. Thus each of the forces in Fig 2.2 is listed as being “mediated” by another new particle; more details about these particles (known as bosons) are listed in Fig 2.3. Associated with each particle and interaction is a “coupling strength” that characterizes the strength with which a particle “feels” the interaction.

Note, for instance, that leptons do not interact via the strong force, and only electrically charged particles interact via the electromagnetic force. Neutrinos, in particular, being uncharged massless<sup>7</sup> leptons, are subject only to the weak force.

The second half of the Standard Model thus appears to be simply a second list of particles, this time of ones that carry forces, along with a set of rules about

<sup>7</sup>Recent observations of the phenomenon of neutrino mixing (see references cited in Ref [4]) make it clear that these particles must necessarily have non-zero mass, but other direct experimental limits suggest they must still be extremely light. Given that gravity will not enter into our discussions again, the claim still stands that neutrinos are subject only to the weak force.

<b>BOSONS</b>			force carriers spin = 0, 1, 2, ...		
Unified Electroweak spin = 1			Strong (color) spin = 1		
Name	Mass GeV/c <sup>2</sup>	Electric charge	Name	Mass GeV/c <sup>2</sup>	Electric charge
$\gamma$ photon	0	0	<b>g</b> gluon	0	0
$W^-$	80.4	-1			
$W^+$	80.4	+1			
$Z^0$	91.187	0			

**Figure 2.3:** The bosons of the Standard Model. Rather than being constituents of matter, these particles are really the agents that relay the fundamental forces between all the various particles in the theory. Figure reproduced by permission [10].

the effects they have on the fermions (and each other). To appreciate the deeper content of the theory, however, we need to pause and review how interactions are introduced in the development of the Standard Model. This will, in turn, give us new insight into the rather stunning success of the theory.

As is clear from the fermion table in Fig 2.1, not all elementary particles are alike. They are distinguished by various characteristics such as mass and electric charge, and the curious label “spin” referred to at the top of the chart. In particle physics, mass is really a derived quantity, something that is determined by all the other characteristics of the particle;<sup>8</sup> these other properties are generally called the particle’s “quantum numbers.” These salient traits summarize (for the physicist) exactly how the particle behaves under any given circumstance. For instance, the spin quantum number describes how the particle reacts to a magnetic field, and the charge conjugation quantum number  $C$  describes the relationship between the particle and its antiparticle. The technical description of the Standard Model thus amounts to an enhanced table of the fermions with many more columns, specifying all of the quantum numbers in explicit detail. With this in mind, let us turn to how the notion of a “gauge charge” for each particle (and with respect to each interaction) arises within the Standard Model.

The incredible theoretical beauty of the Standard Model is that the force-carrying bosons, and the interactions they represent, need not be externally invoked. Instead, they are an almost inevitable result of applying a general<sup>9</sup> principle called *gauge invariance* to the system constructed so far. The forces arise rather

<sup>8</sup>In the full Standard Model, the masses of the fundamental particles is itself generated dynamically, meaning that interaction with yet another force-carrying particle (called the Higgs boson) is what provides a particle with the property called mass.

<sup>9</sup>And considerably sacred

naturally by simply requiring that the fermions “behave reasonably” under certain abstract symmetry operations. This means that aside from the fermion content postulated at the beginning, the entire Standard Model is really a deductive consequence of just a few very natural assumptions. For example, the requirement of electromagnetic gauge invariance amounts to requiring that electrons be insensitive to the local phase of their quantum field; following this statement to its logical conclusion, one finds that an electromagnetic force must exist, and will interact with electrons in a very particular way.<sup>10</sup> This interaction or “coupling term” between the two fields appears with a scaling factor that sets the strength of the interaction. In the case just described, the new term is said to represent a particle of the electron field interacting with a particle of the newly-introduced electromagnetic field, the photon, at some particular point in space and time. The coupling constant turns out to be proportional to the electron charge  $e$ . To complete the introduction of this interaction, every fundamental particle in the theory is then assigned a “charge” that describes its susceptibility to the electromagnetic interaction.

In a similar way, the application of other gauge symmetries leads to the identification of the other fundamental interactions and the assignment of “weak” and “strong” charges to each particle. Thus the final, working version of the Standard Model is a list of the elementary particles, their quantum numbers, and their gauge charges with respect to each interaction.<sup>11</sup> The interactions themselves arise during the construction of the theory itself.

Thus we have a simple recipe for deriving the Standard Model, which schematically can be depicted as follows:

$$((\ell, \nu_\ell)_i, q_j) \oplus (\text{gauge symmetries}) \implies (\gamma, Z, W^\pm, g) \otimes (\text{interaction rules}).$$

The complete Standard Model requires the specification of only 18 parameters on the left-hand-side to fully determine the right-hand-side, from which predictions can be readily made. These parameters amount to basically an enumeration of the coupling strengths of each of the three relevant interactions, the masses of the fermions,<sup>12</sup> and a few additional parameters that turn up in the development of the full theory.<sup>13</sup>

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<sup>10</sup>This description, in short form, is how one arrives at the theory of quantum electrodynamics (QED) from a simple armchair exercise.

<sup>11</sup>Whether one counts gauge charges as quantum numbers is perhaps an issue of semantics rather than physics, for the concepts are quite entangled.

<sup>12</sup>In the dynamic mass generation scheme referred to previously (known as the Higgs mechanism), these masses are replaced by couplings to a new scalar field called the Higgs boson.

<sup>13</sup>One enumeration of the parameters of the Standard Model is as follows: six quark masses; three lepton masses; three couplings for the electromagnetic, weak, and strong forces; four quark-mixing parameters; and the Higgs mass and vacuum expectation value.

This discussion perhaps begins to make clear the widespread appeal of the Standard Model and why the theory as a whole is such an efficient and effective tool: if the inputs are valid, the resulting theory will be *deductively* correct. It *has* to apply to the real, physical world, if we believe in the symmetry properties we’ve imposed. But this same “equation” makes the limitations of the Standard Model framework equally clear. The fermion content of the theory, and the symmetries that are to be respected, are all external inputs. The Standard Model cannot explain *why* the fermions appear with the properties they do, nor *why* the coupling strengths for the different forces fall into the unusual hierarchy that they do. It only offers explanations in terms of its own inputs; it can clearly do no more. Those 18 parametric inputs have to be determined experimentally before any quantitative prediction can result, and those predictions will never explain the seemingly contingent values of those parameters.<sup>14</sup>

The power and limits of the Standard Model as a predictive and explanatory framework should now seem clear. By next understanding the Standard Model as nothing more—and nothing less—than an *effective* field theory, we will see that while it may well be just an approximate description of entirely new physics, it is appropriate in an extremely powerful way for describing *any* phenomena at the distance and energy scales of quarks and leptons. We’ve argued so far that Standard Model has to be correct (given certain assumptions, of course). We now explore the claim that the world couldn’t possibly be any other way—up to a point, quite literally.

### 2.2.3 Effective Field Theories<sup>15</sup>

To the physicist, size is just about everything. The word *scale* is usually preferred, since it captures the broader notion of a typical, natural, or otherwise characteristic dimension associated with some physical process or object. And in physics, scale is intimately connected with the idea of relevance: It is an unspoken physical law that, in most circumstances, the only physical effects that matter in the analysis of some problem are those that are at the same scale as the problem itself.<sup>16</sup> Most physical intuition is really based just on a grasp of what the

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<sup>14</sup>One of the goals of theoretical particle physics is to develop a larger, more inclusive “grand unified theory” that not only unites the three fundamental forces into a single one at sufficiently high energies, but that also predicts or otherwise justifies the values of the parameters of the Standard Model. The task is a challenging one.

<sup>15</sup>While this may appear to be the name of a management seminar for young field theory interns, the word *effective* is used here in the sense of being adequate or sufficient for accomplishing some purpose.

<sup>16</sup>The cases where this isn’t so, such as the study of critical phenomena where short-range interactions can suddenly lead to long-range order, are ever the more interesting because of their exceptional character.

relevant scale for a particular problem or process is. Thus, for instance, though quantum mechanics is required to describe the motions of atoms and molecules, ordinary Newtonian mechanics can be used for automobiles, airplanes, and planets, because the scale of those problems ( $10^{-5}$ – $10^{10}$  m) is so much larger than that of the quantum world ( $10^{-8}$  m and smaller). This notion of “separation of scales” is the same concept at work when we say that Einstein’s special theory of relativity is really only useful at “high speeds.” Here, the scale is the speed of light,  $c$ . When an object’s velocity is within, say, a factor of ten of the speed of light, a proper treatment requires the use of special relativity. At slower speeds, neglecting these corrections will have negligible impact on the result.<sup>17</sup>

In particle physics, these notions take on new importance. Historically, until experiments were developed that showed otherwise, it was completely adequate to speak of atoms as the basic constituents of all matter—until one wanted to explore the interactions between individual atoms, *i.e.* until one started looking at the size scale of the atom itself ( $10^{-10}$  m). Then quantum mechanics is required, and (famously so), the separate notions of “wave” and “particle” become rather contrived and artificial. Similarly, quarks form bound states with sizes of order  $10^{-15}$  m, and masses in the 0.1–1 GeV range; on these scales, the “quark” notion of the Standard Model is exactly the right physical concept for understanding the physics of mesons and baryons.

Conceptually, the idea is that although a different theory may apply at very small distance scales (complete with new particles, forces, or even concepts of space and time), it is possible to describe the long-range effects of that theory with an “effective theory” that captures the dominant low-energy physics to (almost) arbitrary accuracy. The low-energy theory replaces the details of what’s happening on smaller scales with new “effective” interactions that very practically describe the effects that we observe. The new particles or interactions may not correspond to something objectively “real”,<sup>18</sup> but for computational purposes, these low-energy “relics” fully describe the observable physics. At the same time that the effective interactions substitute for the “real” high-energy content in a way that preserves the low-energy physics, they also obscures the details of what’s going on at higher scales. The high-energy degrees of freedom have been basically “integrated out”<sup>19</sup> and so lost, but what remains is a low-energy theory completely and rigorously adequate for making predictions at the low-energy scales. The theory is also left with an implicit cutoff, beyond which the high-energy physics can no longer be

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<sup>17</sup>Some would say that the extent to which this statement that “small effects lead to small corrections” holds is a measure of just how much of the world is linear to first approximation.

<sup>18</sup>We leave it as an exercise for the diligent reader to consider what it means for something to be “real” in this context. To quote the B side of a famous mix tape, “Reality is the only word in the language that should always be used in quotes.”

<sup>19</sup>Because we’re not sensitive to the high-energy degrees of freedom, our measurements essentially average over them.

ignored.

Thus, the constructs we call quarks and leptons and “fundamental” interactions can be considered as low-energy or large-scale approximations to some higher-energy theory. And until we can actually make observations at those scales, we are left with a guessing game, trying to speculate what ultimate (or at least higher-energy) theory might reduce to the Standard Model in the appropriate low-energy limit.

Viewing the Standard Model as an effective theory is appealing in several ways. First, it neatly transfers the burden of explanation to another, more general theory expected to hold at higher energies. As a mere descendant from a more ultimate theory, the Standard Model is not required to “answer” all of the fundamental questions. Secondly, some of the structure of the Standard Model does become more natural against the backdrop of a larger theory [17].<sup>20</sup>

In conclusion, then, we’ve seen that the notion of the Standard Model as a renormalizable, effective field theory really means that it consists of interaction terms that arise naturally from the integration over the additional degrees of freedom of a high-energy theory. From this point of view, the fundamental interactions are simply an insightful identification of terms that would appear in the low-energy approximation to an arbitrary high-energy theory. *Rather than being a description of the contingent way the world happens to be, the Standard Model is a complete description of the way the world must look at relatively small scales, no matter what physics is really present at much smaller ones.*<sup>21</sup>

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<sup>20</sup>For instance, the low dimension terms that appear in the Standard Model Lagrangian are simply those that dominate at scales much less than the scale  $M$  of the high-energy theory. As such, the theory is automatically renormalizable. The appearance of light fermions, and the vector-like and parity-violating couplings of the gauge bosons become natural consequences of chiral and local gauge symmetries of the more primitive theory.

The scheme eventually founders, however, on what is called the “hierarchy problem,” the fact that if the Standard Model is an effective field theory, the Higgs and other scalars naturally should have masses at the scale of the new physics, instead of being light ( $M \sim 1$  TeV) as is currently expected. The best candidates for resolving this and related problems propose new properties such as supersymmetry, compositeness, or large extra dimensions.

<sup>21</sup>A shorter summary might be this: The physics most relevant for describing the phenomena observed at a particular scale is determined by that scale; that is, the constructs we imagine to exist at one scale may just be the low-energy artifacts of a much higher-energy theory.

## 2.3 The $b \rightarrow u \ell \nu$ Decay

We now turn to the  $b \rightarrow u \ell \nu$  decay itself, with an aim to touch lightly on some of the more interesting bits and pieces, rather than provide an authoritative and exhausting review of the literature on this topic.<sup>22</sup> The tour begins with a look at the phenomenon of “quark mixing,” the mechanism behind the  $b \rightarrow u$  transition; this diversion clarifies the role and relevance of the parameter  $|V_{ub}|$ . The discussion then turns to semileptonic  $B$  decay, inclusive versus exclusive measurements, and concludes with a recognition of the experimental “facts of life” that make the study of  $b \rightarrow u \ell \nu$  decays such a challenging and enduring enterprise.

### 2.3.1 Quark Mingling

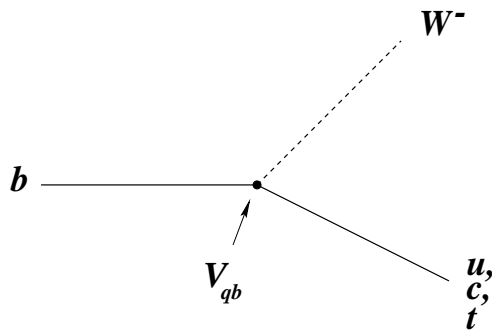
At the heart of the  $b \rightarrow u \ell \nu$  decay is a simple  $b \rightarrow u$  transition. This is an instance of quark “flavor-changing” or “mixing”, where a quark of one flavor<sup>23</sup> transforms into a quark of a different flavor. In the Standard Model, this change of quark flavor is limited to charged weak interactions, *i.e.* those mediated by a charged  $W^\pm$  boson. Fig 2.4 shows the lowest-order charged weak current diagram involving the  $b$  quark.

The mechanism behind this mixing process is the quantum mechanical principle of superposition. The “real” quarks are eigenstates of the Hamiltonian, and are diagonal in the so-called mass basis, but as it turns out, the weak interaction is not diagonal in this representation. The terms in the Standard Model Lagrangian that describe the weak interaction thus introduce a coupling between the different mass eigenstates. As expected in a (complete) linear vector space, however, there exists a unitary transformation that relates the weak and mass eigenbases. By convention, this transformation is expressed as a  $3 \times 3$  unitary matrix called the Cabibbo-Kobayashi-Maskawa (CKM) matrix [18] that operates on the charge  $-e/3$

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<sup>22</sup>For excellent and thorough reviews, see *e.g.* Refs [11, 20, 21, 76].

<sup>23</sup>Recall the discussion of quantum numbers in Sec 2.2.2. For historical reasons, one can also associate with each quark a quantum number measuring the presence of that particle in any other, elementary or not. Thus, for instance, associated with the  $b$  quark is a quantum number that measures the  $b$ -ness of a  $B$  meson or a  $\Lambda_b$  baryon, or even the  $s$  quark (where the assignment would be zero). For even more arcane reasons, these quark-content quantum numbers are said to describe the “flavor” of a particle. Thus a  $b$  quark changing to a  $u$  quark is a change in flavor. (This nomenclature could leave a bad taste in your mouth.)



**Figure 2.4:** The tree-level diagram describing the weak decay of the  $b$  quark. At the weak decay vertex, the  $b$  quark can decay to a  $u$ ,  $c$ , or (kinematically forbidden but possibly virtual)  $t$  quark. In the Standard Model description of the weak interaction, the coupling constant at the vertex is proportional to the the CKM matrix elements  $|V_{qb}|$  ( $q = u, c, t$ ). This matrix is a direct result of the fact that the weak interaction is not diagonal in the quark mass eigenbasis.

quark mass eigenstates  $d_i$  and rotates them to the weak eigenbasis  $d'_i$ :<sup>24</sup>

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.1)$$

### 2.3.2 $|V_{ub}|$

The Standard Model parameter  $|V_{ub}|$  essentially measures the probability amplitude for a lone  $b$  quark to decay into a  $u$  quark mass-eigenstate via the weak interaction. Hence it governs the magnitude of all  $b \rightarrow u$  transitions, and its precise value is important for making predictions within the context of the Standard Model. This property is of course shared generally by all of the CKM matrix elements, but the element  $V_{ub}$  has particular significance. It has been billed as a key ingredient in the incorporation of CP violation into the Standard Model [34] and elsewhere as a “gaping hole” in our understanding [48].

To understand the role of  $V_{ub}$  in the Standard Model, it is useful to explore the structure of the CKM matrix in more detail. There are several possible parameterizations, but here we quote the approximation of Wolfenstein [19], which

<sup>24</sup>The weak interaction terms in the SM Lagrangian have the general form  $\bar{\psi}_i \gamma^\mu (1 - \gamma^5) \psi_i W_\mu$ . For the quark fields,  $\psi_i \equiv (u_i, d'_i)$  for each of the three generations. When the quark mass eigenstates  $q_i$  are introduced, the interaction term breaks into pieces of the form  $\bar{u}_i \gamma^\mu (\sum_j V_{ij} d_j W_\mu)$ , displaying the coupling between each “up-type” quark and all three “down-type” quarks quite clearly.

is sufficient to emphasize the natural (but unexplained) hierarchy of the various elements:

$$V \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (2.2)$$

Here the parameters  $A$ ,  $\rho$ , and  $\eta$  are real numbers intended to be of order unity, and  $\lambda \approx 0.22$  is the “small” expansion parameter. (Historically,  $\lambda$  was originally introduced to explain mixing among only two quark generations, and is given by the sine of the Cabibbo angle,  $\sin \theta_c$ ).  $V_{ub}$  lies in the upper right-hand corner of the matrix, one of only two terms with a complex phase in this representation. This irreducible<sup>25</sup> phase is the sole mechanism by which the Standard Model explains CP violation; the appearance of this complex number in the Lagrangian essentially requires CP violation to occur in order to preserve overall CPT symmetry. In addition, the imaginary part proportional to  $\eta$  directly sets the scale for such violation according to the usual Jarlskög determinant

$$J_{CP} \simeq A^2\eta\lambda^6. \quad (2.3)$$

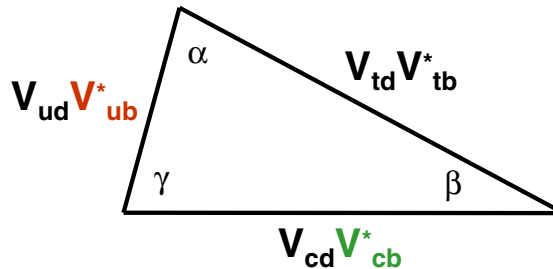
Hence both the magnitude and phase of  $V_{ub}$  are important in constraining the amount of CP violation predicted by the Standard Model.

The CKM matrix must satisfy unitarity by construction. Because of its ubiquitous role in the description of weak decays, there are numerous experimental avenues for measuring most of its elements, either individually or in various combinations. By checking that independent measurements of the various elements are consistent with each other and the overall requirements of unitarity, we probe for inconsistencies in our current understanding of the Standard Model, ultimately hoping to expose new physics processes beyond it.

The familiar “unitarity triangle” is a graphical way of depicting such consistency checks. The unitarity constraints on the CKM matrix are effectively orthogonality conditions on the rows and columns of the matrix, and each constraint can rather naturally be represented as a triangle in the complex plane. Of the six triangles possible, only the one constructed from the  $d$  and  $b$  columns has all three sides of comparable size. If the triangle is appropriately normalized, the CKM element  $V_{cb}$  specifies the length of the base and  $V_{ub}$  determines the location of the apex; see Fig 2.5. Since  $B$  decays are readily accessible to experiment, this triangle is a useful tool for representing theoretical and experimental progress on constraining the elements of the CKM matrix. With the wealth of experimental data now available, the triangle is in fact significantly over-constrained, and provides a powerful method for testing the internal consistency of the Standard Model.

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<sup>25</sup>The fact that there is a complex phase in the matrix is unavoidable if there are at least three quark generations coupled to the charged weak current in the manner previously described (left-handed doublets). Where the phase occurs in the matrix is



**Figure 2.5:** The standard unitarity triangle, a geometric construction representing the requirements of unitarity as applied to the CKM matrix. This is the only one of six possible triangles that has a non-trivial appearance, that is, with all sides of the same order ( $\lambda^3$ ). Note that  $|V_{ub}|$  figures into the height of the triangle.

The measurement of  $|V_{ub}|$  plays an important role in this picture, not because of its direct sensitivity to “new” physics,<sup>26</sup> but because the precision on its measured value affects the utility of independent constraints on the triangle’s other sides and angles. (It is important to note the distinction between an inconsistency in the experimental measurements of the CKM parameters and a true breakdown of unitarity. The latter is unlikely in sensible physical scenarios, but it is possible (even hoped) that conflicting experimental constraints will expose a new sector of physics beyond that described by the Standard Model. Once the new features are incorporated into the theory, and the experimental data is re-interpreted in light of the new physics, however, the consistency of the triangle will be restored. At no point will the true unitarity of the CKM basis transformation be in doubt.)

It is also evident from Eqn. 2.2 that  $|V_{ub}|$  is down by a factor of  $\lambda\sqrt{\rho^2 + \eta^2}$  relative to  $|V_{cb}|$ ; numerically this factor is approximately 0.08, making it an experimental challenge to even measure the decays  $b \rightarrow u$  in the face of much larger  $b \rightarrow c$  backgrounds. This issue of *Cabibbo-suppression* is one of the chief complications to the experimental determination of  $|V_{ub}|$ ; we will return to it again (and again).

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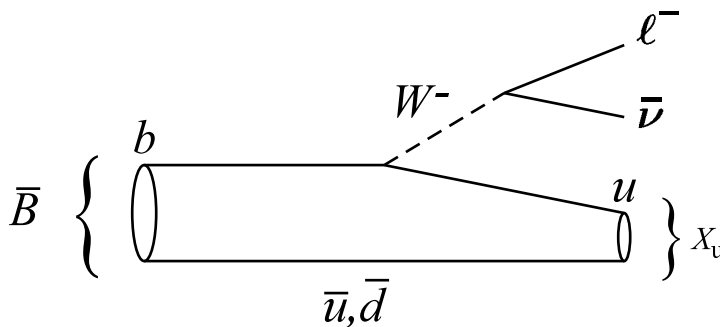
dependent both on the representation (basis) chosen for the matrix and on the order of the approximation, if any is made.

<sup>26</sup>Measurements of  $|V_{ub}|$  can potentially be affected by some types of “new physics,” but the deviations are likely to be small. Physics processes with “loops” in their Feynman diagrams, such as  $b \rightarrow s\gamma$  and neutral  $B$  mixing, are much better candidates for hints of new physics. In a loop, a (virtual) new particle can enter directly, conferring enhanced sensitivity to non-Standard Model particles.

### 2.3.3 Semileptonic $B$ Decay

In a semileptonic  $B$  decay, the  $b$  quark embedded within the  $B$  meson decays weakly to a  $c$  or  $u$  quark, and the virtual  $W$  from that vertex produces a charged-lepton and neutrino pair. The amplitude for the flavor-change  $b \rightarrow q$  is governed by the CKM element  $V_{qb}$ , making the decay rate  $B \rightarrow X_q \ell \nu$  proportional to (among other things)  $|V_{qb}|^2$ . Thus these decays offer direct experimental access to  $V_{cb}$  and  $V_{ub}$ . Furthermore, the relatively large semileptonic branching fraction ( $\Gamma_{\text{SL}} \gtrsim 10\%$ ) guarantees that these decays will be numerous in any sample of  $B$  decays.

Fig 2.6 shows the standard Feynman diagram for semileptonic  $B$  decay. (The presence of both quarks and leptons in the final-state gives this *semileptonic* decay its name.) Note that the partner or “spectator” quark  $q$  ( $= \bar{u}, \bar{d}$  in Fig 2.6) does not participate directly in the weak decay, but must still adjust in the aftermath to the new circumstances of a suddenly different partner quark ( $c$  or  $u$ ). Often, the spectator quark will combine with additional quarks that emerge from the vacuum to make a more complex hadronic system of several mesons.



**Figure 2.6:** Feynman diagram for  $B \rightarrow X_u \ell \nu$  decay. In the more general  $B \rightarrow X \ell \nu$  decay, the final-state  $u$  quark resulting from the weak decay of the  $b$  can be replaced by the (heavier)  $c$  quark.

By collecting all of the final state hadrons into a single hadronic system  $X$ , the decay can be treated as a straightforward 3-body process  $B \rightarrow X \ell \nu$ . Additionally, it is useful to combine the two leptons (charged and neutrino) into the single virtual  $W$  that connects the  $b \rightarrow u$  decay to the production point of the lepton pair. The variable of choice in the decay is then  $q^2$ , the magnitude of the four-momentum transfer to the lepton-neutrino system, also the invariant mass of the virtual  $W$ . Denoting the subsequent hadron as  $X$  with mass  $M_X$ , the following relations follow immediately from considerations of four-momentum conservation:

$$q^2 = M_W^2 = (p_\ell + p_\nu)^2 = M_\ell^2 + 2E_\ell E_\nu - 2|\vec{p}_\nu||\vec{p}_\ell| \cos \theta \quad (2.4)$$

$$= (P - p_X)^2 = M^2 + M_X^2 - 2ME_X. \quad (2.5)$$

Here,  $M$  represents generically the mass of the initial meson, a  $B$  in our case, and  $P$  is its momentum four-vector. Note that the second equation has been explicitly evaluated in the rest frame of the  $B$ , so there is no contribution from its own initial motion. Taking the neutrino mass as vanishing, the expression in terms of the leptonic variables becomes

$$q^2 = M_\ell^2 + 2E_\nu(E_\ell - |\vec{p}_\ell| \cos \theta), \quad (2.6)$$

which is useful when considering the allowed ranges of the various kinematic variables. For instance, one can easily derive the following bounds for  $q^2$  as a function of the observed charged lepton energy  $E_\ell$

$$M_\ell^2 \leq q^2 \leq 2ME_\ell + \frac{2M_x^2 E_\ell}{2E_\ell - M}. \quad (2.7)$$

(Here, we've also taken the lepton  $\ell$  as massless in deriving the upper bound.) It is useful to note here that  $q^2$  attains its global maximum of  $(M - M_X)^2$  when the hadronic decay products are at rest in the  $B$  rest frame and the lepton and neutrino recoil back-to-back with all of the remaining kinetic energy. In this configuration, the probability for the final-state quarks to form a hadron is quite high, and the form factors are at their maximum. In fact, theoretical predictions are often normalized to this point of *zero recoil*, where, to leading order, the form factors are simply unity at  $q_{max}^2$ . Similarly, the form factors reach their minimum when  $q^2$  reaches its global minimum of  $M_\ell^2$ , where the neutrino and lepton are produced collinear with each other and recoil against the final-state quark. Fig 2.7 illustrates these two extreme configurations.

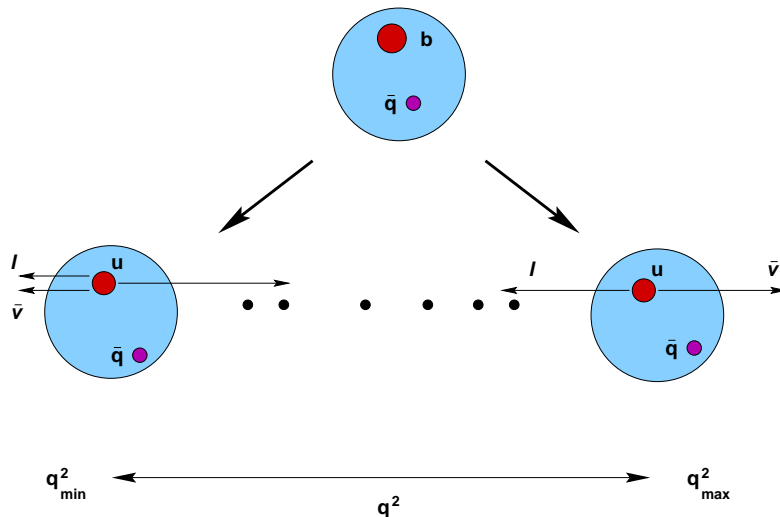
### 2.3.4 A Hard Problem of Soft Gluons

One might expect that the element  $V_{ub}$  could be easily determined from the direct measurement of  $b \rightarrow u \ell \nu$  decays.<sup>27</sup> The nomenclature  $b \rightarrow u$ , however, is slightly misleading, since it really refers only to the quark-level decay that occurs within the strongly-interacting environment of the initial- and final-state mesons. Accounting for the “gluon cloud” and light partner quark that surround the  $b$  quark within the meson is essentially an intractable calculation in QCD. This ever-present collection of low-momentum or “soft” virtual gluons and quarks is often termed the “brown muck” because of the obscuring effect it has on the underlying physics. Peering through this cloud to grasp the weak physics governing the decay is extremely difficult, because the strong physics cannot be simply disentangled from the weak.

Theorists, however, have tackled this dilemma with the introduction of various *models* and the discovery of several *symmetries* that appear in certain limits. The

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<sup>27</sup>This is in contrast to other CKM elements whose values have to be inferred in a less direct manner, from interference or mixing processes, for example.



**Figure 2.7:** An experimentalist’s rendition of the kinematic configurations for the semileptonic decay of a heavy meson such as the  $B$ . The blob at the top represents the meson before decay. The lower figures illustrate the range of final state configurations after the decay  $b \rightarrow uW$ , with  $W \rightarrow \ell\bar{\nu}$ . The kinematic variable  $q^2$  measures the momentum transfer to the lepton-neutrino system, and spans a range from its minimum value when the lepton and neutrino are collinear (depicted on the right side) to its maximum when the  $u$  quark is produced at rest and the lepton and neutrino escape back-to-back (shown on the left). Figure modeled on one found in Ref [21].

semileptonic decay of the  $B$  meson is one area rich with the influx of both types of ideas. The fact that the weak decay of the  $b$  leads to the production of an  $\ell\nu$  pair in the final state (instead of even more strongly-interacting quarks) also helps simplify the theoretical and experimental analysis.

The effects of the strong interaction can usually be disentangled from the well-understood weak physics because the hard parts are limited to the hadronic current, from which the leptonic current completely factorizes. As a result, the effects of the strong interaction can be rigorously parameterized in terms of a small number of *form factors*, which are (unknown) functions of the Lorentz-invariant quantity  $q^2$ . In this sense, the weak and strong physics can be separated, and there is some chance that the dynamics of particular decays can be understood.

For example, the partial width for the exclusive decay  $B \rightarrow \pi\ell\nu$  is easily calculated to leading order as

$$\frac{d\Gamma}{dq^2}(B \rightarrow \pi\ell\nu) = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} p_\pi^3 |f_+(q^2)|^2 \quad (2.8)$$

where  $f_+(q^2)$  is the single form factor that hides the unknown strong physics governing the re-assembly of the  $b$  decay products into a final-state pion. This function,

however, cannot be predicted directly from the theory of QCD; rather, it is either calculated in the context of certain models of quark interactions, or is determined experimentally. Certain theoretical symmetries then relate the form factor for this particular decay to the form factors in other decays, returning some measure of predictive power. Difficulties remain, however, in assessing the “corrections” to the symmetry considerations, and in evaluating the accuracy of a particular model. Nevertheless, with recent progress on model-independent predictions from the lattice, interest in this avenue is surging, as it promises increased precision on both  $|V_{ub}|$  and  $|V_{cb}|$ . For more discussion of these issues, particularly in the context of exclusive decays, consult Ref [76] and the references therein.

### 2.3.5 Inclusive Measurements

Put briefly, an inclusive measurement sums over several decay channels that share some common feature (*e.g.* a global quantum number such as lepton or baryon number). However, both experimentally and theoretically, this formal definition doesn’t reflect how an inclusive analysis is actually performed, nor what the chief advantages of such an approach are.

Experimentally, one abstracts the salient features of the inclusive decay and ignores the particulars of each channel. An inclusive measurement of semileptonic decay typically makes a statement about all  $B$  decays that include leptons in the final state, and all specifics about the hadronic content are ignored. Adopting this technique has consequences on both statistical and systematic uncertainties. Summing over multiple decay channels means a larger data sample, but the lack of a reconstructed decay chain makes it harder to separate signal from background. Writing faithful inclusive Monte Carlo is non-trivial when the “signal” mode is so loosely defined, or equivalently, when many exclusive channels are to be included. Backgrounds have to be well-managed, or at least well-understood, so one is sure of properly selecting the signal modes.

On the theoretical side, an inclusive calculation within the framework of heavy quark effective theory is similarly performed without explicit reference to each possible hadronic resonance. The true benefit of the approach is that that the calculation can actually simplify into one solely at the quark level. The informal equation below captures the spirit of this connection between the physical resonances and the idealized quark-level decay.

$$\sum_i B \rightarrow X_u^i \ell \nu \approx b \rightarrow u \ell \nu \quad (2.9)$$

In essence, the left-hand side is what’s actually observed experimentally, and the right-hand side is the underlying weak decay we’re after. In the limit of quark-hadron duality, *i.e.* when enough exclusive channels are properly included, the irreducible hadronic physics idiosyncratic to each channel becomes irrelevant and

the calculation can be handled at the quark level, albeit with the introduction of a series of effective local interactions and a collection of non-perturbative parameters. These parameters, which arise even at leading order (namely, the quark mass  $m_b$ ), are to some extent universal across  $B$  decays, however, so a predictive framework is still possible. Essentially, the final-state hadronic physics is averaged out, leaving behind the bound-state effects in the  $B$  meson, but these can be abstracted and determined from other decays.<sup>28</sup>

Generally, inclusive measurements of decay rates help confirm our understanding of total widths and their decomposition into exclusive modes, and so can also be sensitive to new physics [28]. But the tradeoff between an inclusive and exclusive measurement is by no means clear-cut. The experimental measurement may be easier to implement mechanically and may be subject to different (sometimes independent) sources of error, but it can still require detailed understanding of backgrounds and detector effects. The theoretical prediction benefits from a firm connection to the underlying theory, and can, to a large extent, sidestep the problem of bound-state effects, without reference to a particular model. But the inclusive theory can only make statements about an idealized decay in the limit of quark-hadron duality. And, as we shall see in the sequel to this section, experimental realities can restrict the inclusive calculation in ways that can lead the theory to actually break down, leaving theorists to resort to other, less rigorous tools. Ultimately, progress on  $|V_{ub}|$  requires active pursuit along all possible avenues.

### 2.3.6 In a Nutshell: The Challenge of $b \rightarrow u \ell \nu$

Inclusive study of the semileptonic decay  $b \rightarrow u \ell \nu$  requires forging a careful compromise between the twin standards of particle physics analysis: high efficiency and low fake rates. For statistical reasons, it is vital to collect as many events as possible, but to ensure a high-quality result, it is equally essential to make sure that the event sample includes *only* what's intended. Balancing these two ideals is the general experimental challenge.

The snag in the experimental pursuit of  $b \rightarrow u \ell \nu$  is singling out the semileptonic  $b \rightarrow u$  decays from the numerous other  $B$  decay modes. Although the semileptonic decay route is one of the more common ones (the branching fraction is about 10.2% [5]), restricting interest to the *charmless* decays  $B \rightarrow X_u \ell \nu$  is what makes the analysis difficult. The problem is that  $b \rightarrow c$  transitions overwhelm  $b \rightarrow u$  ones by an order of magnitude,<sup>29</sup> so tight experimental cuts are

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<sup>28</sup>The non-perturbative parameters must be determined in a sensible and consistent renormalization scheme in order to be of application to another decay. Measurements made in one process cannot be applied to another blindly.

<sup>29</sup>The extent to which the charm contribution dominates depends on the range where one looks. As a whole, the charmless branching fraction is less than 5% of the total

necessary to eliminate the former decays from the event sample. The kinematics of the two semileptonic channels are similar, so most experimentally-accessible variables offer little discrimination—except near the ends of their allowed ranges, where the mass difference between the  $c$  and  $u$  quarks can become important. The experimental challenge is thus finding—and applying—a discriminating variable (or set of variables) that reliably distinguishes the  $b \rightarrow u$  decays from  $b \rightarrow c$  and that has a ready theoretical interpretation.

One historically popular technique for distinguishing  $b \rightarrow u \ell \nu$  employs a cut on the energy of the charged lepton  $\ell$ . While this is no longer the only avenue used to isolate  $b \rightarrow u \ell \nu$  decays, it captures the essential difficulties that plague most approaches. In the  $B$  rest frame, the kinematic endpoint of the lepton energy spectrum can be calculated by considering the case where the (massless) neutrino is produced at rest and the lepton and hadronic system recoil against each other with maximum energy. In this limit, the decay can be treated essentially as a 2-body process, kinematically-speaking. The lepton energy in this circumstance is given by:

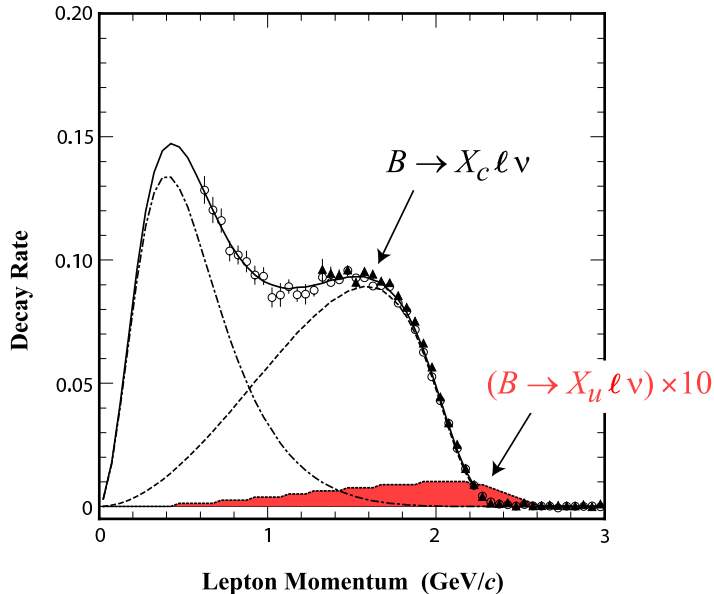
$$E_\ell = (M_B^2 + M_\ell^2 - M_X^2)/(2M_B). \quad (2.10)$$

Since the lightest  $X_c$  meson (the  $D$  at 1.87 GeV) is heavier than the lightest  $X_u$  meson (the  $\pi$  at 0.140 GeV), it is clear that the lepton energy spectrum for charmless decay extends beyond that for charm. Numerically, the endpoint for  $B \rightarrow D \ell \nu$  is 2.31 GeV, while for  $B \rightarrow \pi \ell \nu$ , it is higher at 2.69 GeV. The idea behind the experimental cut is that by looking for leptons with energies above the kinematic limit for  $b \rightarrow c$ , one can be confident that the decay is genuine  $b \rightarrow u$ . As Fig 2.8 shows, however, finding these high-momentum leptons is hard because they're rare, and measuring their momentum precisely is critical to ensuring they're true signal. With an overwhelming  $b \rightarrow c \ell \nu$  rate sitting just below the charm endpoint, even relatively small or infrequent measurement errors can cause the  $b \rightarrow u \ell \nu$  signal region to be flooded by mismeasured  $b \rightarrow c \ell \nu$ . This situation, where small measurement errors can cause the ever-present  $b \rightarrow c \ell \nu$  background to smear into an ideally background-free region and swamp the  $b \rightarrow u \ell \nu$  signal, is a classic example of the difficulties on the experimental front. But there is a far more serious obstacle on the theoretical side.

As just noted, the kinematic window for cleanly observing  $b \rightarrow u \ell \nu$  is only a few hundred MeV wide—a tiny slice of a spectrum that covers a range an order of magnitude larger—so only a small fraction of the total  $b \rightarrow u \ell \nu$  rate is sampled above a cut at the endpoint for decays to charm. In order to extrapolate measurements from this region to the full charmless semileptonic rate, or to interpret even the partial rate in the context of some theoretical expectation, a detailed understanding of the spectrum in the region of the kinematic endpoint is necessary. But the physics in this small window above the charm endpoint is controlled by

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semileptonic fraction, but in the region above the kinematic endpoint for charm,  $b \rightarrow u$  is the only contributor, up to leakage from  $B \rightarrow X_c \ell \nu$  due to detector smearing or bias.



**Figure 2.8:** The lepton momentum spectrum from  $B$  decays as observed at CLEO. The points indicate the measured decay rate in each momentum bin, open circles for electrons, and solid ones for muons. The dashed curve shows the contribution expected from the dominant  $b \rightarrow c \ell \nu$  process, and the shaded area at the bottom represents the contribution from the much rarer  $b \rightarrow u \ell \nu$  process, which has been increased by a factor of ten for display purposes. The dash-dotted curve shows the contribution from so-called “secondary” leptons produced in the sequential “cascade” chain  $b \rightarrow c \rightarrow s \ell \nu$ . The solid curve represents the mathematical sum of all three sources of leptons, and agrees well with the measured spectrum. Figure modified by the author from one that originally appeared in Ref [11].

the same incalculable strong dynamics that binds the  $b$  quark into the  $B$  meson. To see this clearly, note that Eqn 2.10 reveals that the endpoint for the free quark decay  $b \rightarrow u \ell \nu$  is about  $m_b/2 \approx 2.3$  GeV (using  $m_b \sim 4.65$  GeV), coinciding with the charm endpoint, while the physical endpoint is at  $M_B/2 \approx 2.6$  GeV (using  $M_B = 5.3$  GeV). It is *entirely* due to the fact that the  $b$  quark is embedded in the  $B$  meson that the charmless endpoint is pushed up to 2.6 GeV.<sup>30</sup> But the details of how the  $b$  quark behaves inside the  $B$  meson are fundamentally non-perturbative, and cannot be calculated with the traditional tools of perturbative field theory. So

<sup>30</sup>Another way of seeing this same problem is to note that Eqn 2.10 implies that the change in the lepton endpoint varies with parent mass  $M$  according to  $\Delta E_\ell \approx M/2$ . By changing from an idealized quark-level description with  $M \rightarrow m_b = 4.65$  GeV to the actual meson-level description with  $M \rightarrow M_B = 5.3$  GeV, the endpoint is shifted higher by 350 MeV, precisely overlapping the charm-free window so critical to experimental identification of the decay as  $b \rightarrow u \ell \nu$ .

prediction of the lepton energy spectrum in this region is out of theoretical reach, dictated by the inaccessible details of the  $b$  quark motion in the meson.

Thus we are stuck between a rock and a hard place: The lepton energy cut required for experimental identification of the  $b \rightarrow u \ell \nu$  signal restricts us to a region of  $b \rightarrow u \ell \nu$  phase space where there can be no easy theoretical prediction or even interpretation, essentially orphaning the measurement. But how else to get  $|V_{ub}|$ ?

The lepton energy cut is but one possible experimental technique for isolating  $b \rightarrow u \ell \nu$ . Cuts on other kinematic variables are possible, with different sacrifices in efficiency and theoretical adequacy. But the interplay between theory and experiment that this example illustrates is typical of almost any analysis of  $b \rightarrow u \ell \nu$  decay.